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TWO-DIMENSIONAL CRITICALITY CALCULATIONS OF GASEOUS-CORE CYLINDRICAL-CAVITY REACTORS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

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The critical mass of a cylindrical, gaseous-core cavity reactor system was computed with a four-group, two-dimensional diffusion code. Uranium 235 and plutonium 239 were considered as fuels. Moderator materials investigated were deuterium oxide (at 70° F) and graphite (at 70° and 5300° F). The study was conducted in two parts; the first part considered the effect on critical mass of variable fuel-region radius in a fixed-size cavity, and the second part pertained to specific effects for one particular configuration.

The fuel region can be reduced to a radius of 0.20 to 0.33 of the cavity radius with little or no increase in critical mass. For fuel- to cavity-radius ratios from 0.20 to 1.0 and a graphite moderator at 5300° F, plutonium 239 requires about one-third of the critical mass of a uranium 235 fueled system. A hole (simulating an exhaust nozzle) through one end of the reflector-moderator region resulted in an increase in critical mass of 5 percent. The hydrogen gas, which in nuclear-rocket application would exist between the fuel and moderator regions, was found to have a negligible effect on critical mass. The critical mass is relatively independent of fuel-region temperature and can be computed by evaluating all thermal values at the moderator temperature.

INTRODUCTION

To obtain nuclear-rocket propellant temperatures and specific impulses above the limits now imposed by solid fuel elements, gaseous-fueled reactors may be used. In the last few years there have been several such reactors proposed. A gaseous-fueled reactor that utilizes the flow of a fuel-propellant mixture through a number of vortex tubes having large ratios of length to diameter is discussed in reference 1. Reference 2 contains a commentary on a reactor that allows the fuel and propellant to enter in separate coaxial streams with the fuel flowing at low velocities relative to the propellant (fig. 1(a)). Both of these suggestions necessitate the use of hydrodynamic forces to separate the fuel from the propellant gas. A third method (refs. 3 and 4) employs magnetic forces to keep the fuel confined to a particular region. All three of these plans utilize a gaseous fuel that is in a cavity region surrounded by a reflector-moderator region. The last two suggestions, coaxial and magnetic containment of the fuel, require that the fuel be in a central region relative to the cavity boundary.

The nuclear calculations presented in reference 5 are given for spherical geometries in which the cavity is uniformly filled with a nuclear fuel. In some of the cavity-reactor methods proposed for nuclear-rocket application, there are regions between the moderator and the fuel, referred to herein as propellant flow regions, through which neutrons may pass without entering the fuel region. Therefore, results for this method might indicate increases in critical mass compared with the results of reference 5. Some analytical results for spherical cavities with variable fuel-region radii within the cavity are reported in reference 6. The results presented herein are an extension of this work to cylindrical-cavity reactors. Included in the present study are the effects on critical mass of (1) variable fuel-region radii being within two cavity radii (150 and 40 cm) for cavity length to diameter ratios of 1 and 2, respectively, (2) replacing the deuterium oxide (D_2O) reflector-moderator with graphite (C), (3) increasing the graphite moderator temperature from 70° to 5300° F, which bounds the range of interest, and (4) replacing uranium 235 (U^{235}) fuel with plutonium 239 (Pu^{239}). Also included are indications of temperature changes in fuel and moderator regions and changes in critical mass caused by the addition of hydrogen (H_2) in the propellant flow region and by the opening in one end of the moderator region, which simulates an exit nozzle. The results presented here were obtained through the use of PDQ02 (ref. 7), a four-energy group, two-dimensional (r,z) diffusion code for the IBM 704 computer.

SYMBOLS

A atomic mass

D cavity diameter

E neutron energy

E_o neutron resonance energy

g $\frac{1}{2}\left(\frac{2J + 1}{2I + 1}\right)$

I spin of target nucleus

J spin of compound nucleus

k Boltzmann's constant

L cavity length

M molecular weight

n number of neutrons per cubic centimeter of energy E per unit energy interval

r radial dimension

r_i radius of fuel region

r_o radius of cavity region

T material temperature

u axial velocity

v neutron velocity

w mass flow

z axial dimension

Γ level width

ν neutrons produced per fission

σ microscopic cross section

φ neutron flux

Subscripts:

a absorption

F nuclear fuel

f fission

H_2 hydrogen

n neutron

R removal

s scattering

th thermal

γ radiative capture

Superscript:

$-$ average

REACTOR ANALYSIS

Geometry

The results presented in reference 6 are basically for a spherical reactor configuration. A more applicable geometry for nuclear-rocket use is the cylindrical geometry. The configuration shown in figure 1(b) is that of a right cylindrical reactor. In all the reactors presented, the reflector-moderator region was maintained at a thickness of 100 centimeters, approximately the optimum thickness (ref. 6) for reducing critical mass without incurring excessive total weights. The moderator materials selected for computation were heavy water (100 percent D_2O) and graphite. The thermal neutron energies used corresponded to the moderator temperatures of 70° F for D_2O, and 70° and 5300° F for graphite. The fuel region with radius r_i was centrally located in the cavity region (with radius r_o) and extended the entire length of the cavity. This geometry most nearly corresponds to a coaxial flow reactor system (fig. 1(a)). The major portion of the calculations involved varying the fuel-region radius within the cavity and determining the critical mass.

Inasmuch as the weight of a cavity reactor is primarily that of the reflector-moderator, which is the outer region, the dimensions of the cavity must be adjusted to keep the weight down, but at the same time the propellant flow area must be sufficient to satisfy thrust requirements. The combined continuity equations for the propellant (H_2) and fuel (U^{235} or Pu^{239}) were used to determine a reasonable fuel- to cavity-radius ratio r_i/r_o. The equation, expressed as an axial velocity ratio of hydrogen to fuel, is as follows:

$$\frac{u_{H_2}}{u_F} = \frac{\left(\frac{w_{H_2}}{w_F}\right)\left(\frac{M_F}{M_{H_2}}\right)\left(\frac{\overline{T}_{H_2}}{\overline{T}_F}\right)}{\left[\frac{1}{\left(\frac{r_i}{r_o}\right)}\right]^2 - 1}$$

The assumptions and restrictions applied to this equation are:

(1) Based on hydrodynamic considerations, the average axial velocity ratio u_{H_2}/u_F must be on the order of 100 or less.

(2) A mass-flow ratio w_{H_2}/w_F of 100 was used as an economical limit for nuclear fuel consumption.

(3) Based on present reasonable design considerations, the maximum cavity pressure is 5000 pounds per square inch.

(4) As estimated from radiation heat-transfer considerations, the average hydrogen- to fuel-temperature ratio $\overline{T}_{H_2}/\overline{T}_F$ is approximately 0.5.

With these restrictions, a cavity radius of 150 centimeters was chosen as a representative value. For some of the calculations a fuel-region radius of 25 centimeters was chosen to give a ratio of practical interest. To determine the effect on variable fuel-region radii within the cavity, cavity radii of 150 and 40 centimeters were selected.

Two-Dimensional Diffusion

A reactor code (ref. 7) that solves the few-group neutron-diffusion equations for four lethargy groups was used to determine neutron flux distributions and critical masses for a two-dimensional (r,z) geometry. The configuration selected was a cylinder composed of a 25-by-25 mesh. This was a compromise between computing time and improved accuracy. By using a 50-by-50 mesh, the particular machine limit, the results were improved by approximately $2\frac{1}{2}$ percent in reactivity (decrease). The four energy groups divided the neutron energy spectrum in the following manner. Group one, the fast group, was for energies between 10 and 1.5 million electron volts (Mev); group two was for those between 1.5 and 0.0043 Mev; the third group was for those between 0.0043 Mev and 5 kT electron volts (ev). The material temperatures used for the thermal neutron energy group were either 70° or 5300° F. Microscopic cross sections representing average values for the four energy groups used are given in table I. The average cross sections were obtained by a flux weighting method used in reference 8.

Thermal cross sections for uranium 235 and plutonium 239 were obtained as described in the appendix. The microscopic cross sections are obtained first as a function of neutron energy for a range of fuel temperatures. As the fuel temperature increases, the Doppler broadening effect broadens the width and decreases the peaks of low-energy resonances. The averaged thermal cross sections are then obtained by using Maxwellian distributions based on moderator temperature.

For the reactors in which the fuel region does not fill the cavity, hydrogen was used for the propellant region between the fuel and the inner reflector region. The hydrogen density was assumed constant at 0.8×10^{21} atoms per cubic centimeter based on estimated temperature and present conditions.

For large cavities (greater than 60 cm in radius), the computing time for the PDQ02 program was prohibitively long. For example, the case for a cavity radius of 40 centimeters and a cavity length to diameter ratio L/D of 1 required 3 hours to compute; the 60-centimeter case required approximately 9 hours of machine time. An estimate of running time for an 80-centimeter-radius cavity indicated that approximately 40 to 60 hours of machine time might be required. Therefore, extrapolations were applied to estimate critical mass for uniformly filled cavity reactors. In addition, a buckling analogy was used for comparison with extrapolations for the D_2O reflected curves of uranium mass plotted as functions of the fuel- to cavity-radius ratio.

Buckling Analogy

In order to furnish a comparison of extrapolated critical mass for a cavity uniformally filled with uranium, an analogy from reference 6 was used for D_2O moderated-reflected reactors. The analogy results from equating geometric buckling for cylinders to spherical buckling. The fuel density is assumed to be equal for both geometries, and the spherical buckling is obtained from one-dimensional calculations. By equating the cylindrical geometric buckling to the spherical buckling, the dimensions of the cylinder can be obtained. By equating the fuel densities, the critical mass can be determined. In the analogy the assumption is made that the correct radius at which to equate the buckling is between the cavity radius and the radius of the reactor.

RESULTS AND DISCUSSION

The basic configuration studied is shown in figure 1(b). In this figure, the fuel and cavity regions are completely enclosed by a reflector-moderator. For the calculations made here, this reflector-moderator region is either D_2O or graphite and is 100 centimeters thick. The fuel region of radius r_i, centrally located with respect to the radius, is allowed to vary in radius inside a cavity of radius r_o. The majority of calculations were performed with a cavity L/D of 1; a few were performed for an L/D of 2.

As mentioned in the section on REACTOR ANALYSIS, the coaxial flow reactors of practical interest are those with small fuel- to cavity-radius ratios. Of academic interest for this reactor concept is the effect of fuel- to cavity-radius ratio on critical mass for the entire range. The extrapolated values for cavities uniformly filled with uranium fuel were compared with results obtained from figure 2. This set of curves represents a range of critical masses for L/D values of 1 and 2 obtained by equating spherical buckling to cylindrical geometric buckling in the same manner as presented in reference 6. Another method that yields similar results would be that of equating cavity volumes. The upper curves for each L/D were obtained when the radii in the buckling equations were assumed to be the cavity radius, and the lower curves for each L/D were obtained by assuming the radii were the radii of the cavity plus the reflector thickness. The PDQ02 results shown in figure 2 fall between these limits. The extrapolations to cavities uniformly filled with uranium fuel were then made by using the curves. To be conservative, the upper limit was used for the extrapolation.

Effect of Variable Fuel-Region Radii

In the spherical results presented in reference 6, a fuel- to cavity-radius ratio of approximately 0.5 was reached before an additional 10 percent uranium fuel was required. In a cylindrical coaxial flow reactor, the fuel is assumed to extend the full length of the cavity so that only the fuel radius is reduced. In figure 3 results for several different configurations are presented. The bottom curve is for a small cavity radius (40 cm). It is shown to illustrate a completely calculated curve with a variable fuel radius and to indicate the trend for the larger cavities. For this particular reactor the fuel- to cavity-radius ratio was reduced to approximately 0.2 before the critical mass began to increase noticeably.

For the three curves in figure 3 with a cavity radius of 150 centimeters, the reactors were too large to obtain calculated results in reasonable lengths of time. For the D_2O reflected reactors, the critical masses were extrapolated by using results from figure 2, and for the graphite reflected reactors the extrapolations (dashed portions) were made by assuming the same shape as the D_2O curves beyond the calculated values indicated by the solid portions. All the curves in figure 3 indicate that the fuel-region radius can be reduced to approximately one-third of the cavity radius before an increase in critical mass is required. The D_2O (70° F) moderated cavities indicate that the fuel-region radii can be relatively smaller than with the graphite (5300° F) moderated cavities.

One point of interest for the plutonium fueled reactors is that the curve begins to increase more sharply than the comparable uranium fueled, graphite reflected reactors, even though the critical mass and density are lower. This is primarily due to the increased self-shielding of plutonium resulting from the absorption cross sections (see table I). Of more importance, figure 3 also shows that the use of plutonium 239 will reduce the critical mass (and therefore reactor pressure) required for criticality for radius ratios between 1.0 and 0.16.

As the fuel-region radius decreases, the fuel density increases, which causes the macroscopic absorption cross section to increase. This increases the self shielding of the fuel region, which is reflected in the thermal flux as shown in figure 4. Plotted in figure 4 for a cavity radius of 150 centimeters are the normalized thermal fluxes for three different fuel-region radii. As the fuel-region radius decreases from 40 centimeters to 25 and 15 centimeters, the fuel density required to maintain a constant criticality factor increases from 0.97×10^{19} to 2.88×10^{19} and 11.31×10^{19} atoms per cubic centimeter, respectively. With these increases in fuel density, the flux plot shows the increase in the self-shielding effect. If the fuel had been plutonium, this sharp drop in thermal flux through the fuel region would have occurred at a larger radius ratio because of its larger absorption probability, as previously indicated.

Effect of Exhaust Nozzle

For the cavity reactor especially, an exhaust nozzle through the reflector region represents a direct exit for neutrons from the reactor. It is of interest to determine the effect on critical mass for an exhaust nozzle, or hole, through one end of the reflector-moderator. The hole was inserted as shown in the cross-sectional view in figure 5(a). The diameter of the hole was equal to the diameter of the fuel region, 50 centimeters. The critical mass from figure 3 for a D_2O reflected cavity with no exhaust hole, $L/D = 1$, a cavity radius of 150 centimeters, and a fuel-region radius of 25 centimeters was approximately 6.3 kilograms.

For the configuration with the end hole, the critical mass increased from 6.3 to 6.6 kilograms. The flux plots of radial and axial flux are shown in figure 5. The peaking of the flux in the D_2O region appears to occur closer to the cavity (~12 cm D_2O) in the radial geometry than in the end reflector region (~25 cm D_2O). The flux in the axial direction also peaks to a higher value than does the radial neutron thermal flux. These conditions are due to the axial flux being plotted along the center axis so that the end reflector region sees the entire length of the fuel region or a high fast flux.

In figure 5(b) the axial thermal flux through the void region simulating a nozzle opening indicates no peaking of flux as compared with the flux at the other end with no hole. A neutron balance based on 1000 neutrons is shown for the reactor with and without an end hole in table II. The value for neutron loss due to leakage in the reflector has been increased by four of every 1000 neutrons when compared with the case with no nozzle in table II(b). Inasmuch as this result is only for one configuration, it is inconclusive, but the indication is that the leakage through an exhaust nozzle will not result in a severe increase of critical mass requirement.

Effect of Replacing Deuterium Oxide by Graphite

Because of the neutron and gamma heating in the reflector-moderator region, cooling requirements may indicate the use of a reflector-moderator material of higher operating temperature capability. Since D_2O does not have high-temperature capability, graphite was also considered. In the same configuration as in the previous section (without the nozzle opening) the D_2O is replaced by graphite. Because of the poorer moderating properties of room-temperature graphite, an increase in critical mass was expected.

For graphite at 70° F the critical mass increased to 65 kilograms of uranium 235 from 6.3 kilograms for D_2O. This increase in critical mass is caused mainly by the increased neutron absorption in the graphite. A neutron balance indicating the sharp increase in neutron absorption by the reflector region is shown in table III(a). There are almost five times as many neutrons absorbed in graphite as were absorbed in D_2O (table II(b)). The comparison of tables also indicates a shift in the energy spectrum for fission absorption due to self-shielding and increased fuel density. In the D_2O reflected cavity approximately 99 percent of the neutrons absorbed for fissioning are absorbed in the thermal group, while approximately 90 percent are absorbed thermally in the graphite reflected cavity.

The thermal radial flux shown in figure 6 for the 70° F graphite again illustrates the self-shielding effect of the fuel region. Although there was a factor of 5 in the increase in absorptions in the reflector-moderator region, the inclusion of the self-shielding effect of the fuel increased the uranium required for criticality by a factor of 10 as compared with the D_2O reflected cavity reactor.

Effect of Moderator Temperature on Critical Mass

Since the absorptions in the graphite were the primary cause for the increase in critical mass, increasing the temperature of graphite (a $1/v$ absorber) and hence neutron temperature will reduce the required critical mass provided the decrease in the absorption probability in uranium is not too large. Actually the decrease in absorption probability in uranium as the thermal neutron temperature increases is not as detrimental as might be expected. As the cross section decreases, the absolute value of the thermal flux gradient decreases.

Therefore, the self-shielding decreases, and this reduces the effect on critical mass associated with the decreasing absorption probability in the fuel.

Again for the same size reactor (100-cm reflector, 150-cm cavity radius, 25-cm fuel-region radius, and a cavity L/D of 1), a critical mass of 38 kilograms was calculated for the case of 5300° F graphite. Compared with graphite at 70° F, which required 65 kilograms, the increase in temperature does reduce the absorptions in graphite more than enough to compensate for the reduced absorption probability of uranium. The flux plot in figure 6 for 5300° and 70° F graphite illustrates the change in the flux gradient in the fuel region. The change in the flux gradient from the 70° F case to the 5300° F reflector is due to the decrease in both the critical mass and the absorption probability in uranium. The reduction in neutron absorption by the graphite is shown in table III(b). The comparison of tables III(a) and (b) shows that the absorptions per thousand neutrons has decreased to 144 in the 5300° F graphite as compared with 318 in the 70° F graphite.

Effect of Using Plutonium 239 as Nuclear Fuel

The thermal microscopic cross sections listed in table I for plutonium and uranium indicate that plutonium 239 has a much higher absorption and fission probability than uranium 235. With the same geometry configuration used in the previous example, replacing uranium with plutonium 239 gives a calculated critical mass of 27 kilograms. This is a reduction in critical mass from the uranium fueled cavity with the high temperature graphite that required 38 kilograms.

At 5300° F the microscopic absorption cross section in the plutonium is approximately six times the microscopic absorption cross section in uranium for the thermal group. Since the fuel decreases from 38 to 27 kilograms, the thermal macroscopic absorption cross section for plutonium 239 is still several times larger than for uranium 235. The thermal neutron flux gradients in the fuel region should therefore be much steeper than for uranium 235. This is illustrated in figure 7. The flux falls off very sharply through the fuel region for the plutonium case because of the much higher absorption probability, even though the fuel density is lower than for the uranium fueled cavity reactor. The comparison of the absorptions in the fuel region given in tables III(b) and IV illustrate this increase in neutron absorption with plutonium. This again suggests that plutonium fueled cavity reactors may encounter an uncertain region, where critical masses increase rapidly as the fuel-region radius is decreased, sooner than the uranium fueled reactors, as indicated in figure 3. There may, therefore, be a range at the low values of radius ratio (below 0.16) where uranium 235 might be just as good as plutonium, if not better. For the graphite (5300° F) moderated cavities with $r_o = 150$ centimeters and $r_i = 25$ centimeters, it appears that plutonium 239 is better than uranium 235 between the radius ratios of 1.0 and 0.16. The aforementioned effects are summarized in the following table:

Reflector	Fuel	Temperature, °F	Nozzle	Critical mass, kg
D_2O	U^{235}	70	No	6.3
D_2O	U^{235}	70	Yes	6.6
C	U^{235}	70	No	65.0
C	U^{235}	5300	No	38.0
C	Pu^{239}	5300	No	27.0

Effect of Hydrogen in Cavity

For all the preceding calculations, σ_R and σ_a of the hydrogen gas between the fuel and moderator regions were taken as zero. To check this assumption, absorption and removal cross sections of hydrogen were included for one case. For this calculation, the hydrogen atom density was taken to be two times that of the uranium on the assumption that the average fuel temperature would be twice that of the propellant. For a U^{235}/D_2O system, with $r_o = 150$ centimeters, $r_1 = 25$ centimeters, and $L/D = 1$, the effect of the hydrogen decreased the reactivity by less than 2 percent. The conclusion is that absorption and slowing down effects due to the hydrogen are negligible.

Effect of Fuel-Region Temperature

Gaseous-core cavity reactors have the relatively unique characteristic that the fuel region would operate at a temperature considerably higher than that of the moderator region. For a nuclear-rocket engine application, for example, the fuel core could be in the range 5000° to 10,000° K, while the moderator-region temperature would be limited to 2000° to 2500° K. A brief calculation was made of this effect and is described in the appendix.

Microscopic absorption and fission cross sections of uranium 235 and plutonium 239 were calculated as a function of thermal (moderator) neutron temperature and fuel temperature. These results are shown in figure 8. For a range of thermal neutron energies from 0.025 to 1.0, the average fuel cross sections are essentially invariant with fuel temperature in the range 0° to 10,000° K. It appears that gaseous-core cavity reactor calculations can be carried out with the assumption that the higher temperatures in the fuel region have little effect on reactivity and that the thermal neutron temperature can be assumed that of the moderator region.

CONCLUDING REMARKS

It appears necessary to use a graphite reflector-moderator in order to obtain the highest possible gas temperature. Because of the manner in which gamma and neutron heating decrease in the reflector, it may be desirable to replace the outer portion of the graphite with other reflector materials such as beryllium,

beryllium oxide, and/or deuterium oxide (D_2O) to help reduce the critical mass and reactor weight.

In a reactor such as the gaseous-core cavity reactor, where there are large flux gradients near boundaries, the accuracy of diffusion theory must be considered. Unfortunately, there are no experimental data for gaseous-fuel zones of varying radii within an externally moderated cavity. In reference 9, however, an experiment was conducted on a D_2O moderated-reflected cavity reactor in which the uranium was in foil form lining the outer portion of the cylindrical cavity. By using diffusion theory a spherical calculation was performed that assumed the same surface area as in the cylindrical geometry. The calculation predicted a static criticality factor approximately 9 percent too high. In this case diffusion theory predicted a critical mass about 14 to 15 percent too low, but since the experiment is only similar in that the neutrons are thermalized in the D_2O region, this yields no general conclusion in regard to gaseous-core reactor calculations.

In regard to reactors with strong flux gradients, an effect on the neutron spectrum seen by the nuclear fuel would exist. In the analysis a Maxwellian distribution was used to determine the average thermal cross sections. As the fuel region is decreased in diameter and the fuel density is increased, the hardening of the neutron spectrum becomes more important in determining cross sections. At present this effect appears to require a study of its own and is beyond the scope of this report.

SUMMARY OF RESULTS

Based on diffusion theory and the assumptions presented, including those in the buckling analogy and in the temperature dependence of microscopic cross sections, critical masses of a series of two-dimensional, gaseous-core cavity reactors were computed, and various effects were investigated for one configuration.

For a cavity radius of 150 centimeters, a cavity length to diameter ratio of 1, and a reflector thickness of 100 centimeters, the following results were obtained:

1. The critical mass remained approximately constant for fuel- to cavity-radius ratios from 1.0 to at least 0.33 for either deuterium oxide (D_2O) or graphite reflectors and either plutonium 239 (Pu^{239}) or uranium 235 (U^{235}) fuel.

2. For a reactor that uses a D_2O reflector and uranium fuel, the fuel- to cavity-radius ratio can be decreased to approximately 0.2 before the critical mass increases.

3. The use of plutonium 239 reduces the critical mass by about a factor of 3 compared with uranium 235 with graphite reflectors for a range of fuel- to cavity-radius ratios of 1.0 to 0.30.

For a reactor with a fixed fuel-region radius of 25 centimeters inside a cavity with a radius of 150 centimeters and the same dimensions as the aforementioned reactor, the following results were obtained:

1. A hole 50 centimeters in diameter through one end of a D_2O reflector increased the critical mass of uranium 235 from 6.3 to 6.6 kilograms or about 5 percent.

2. Replacing the D_2O reflector-moderator with graphite at 70° F increased the required critical mass from 6.3 to 65 kilograms.

3. Raising the temperature of graphite to 5300° F reduced the critical mass of uranium 235 from 65 to 38 kilograms.

4. Although the critical mass of plutonium 239 is less than that of uranium 235 for fuel- to cavity-radius ratios greater than 0.20, the higher absorption cross section of plutonium 239 results in greater self-shielding. For fuel- to cavity-radius ratios from 0.30 down to 0.16 this effect begins to diminish the advantage of plutonium 239 over uranium 239.

5. The effect of hydrogen in the propellant region appears to be negligible.

6. For a given geometry and materials, critical mass is relatively independent of fuel-region temperature.

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National Aeronautics and Space Administration
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APPENDIX - TEMPERATURE DEPENDENCE OF

MICROSCOPIC FUEL CROSS SECTION

To obtain Maxwellian averaged microscopic neutron capture and fission cross sections as functions of thermal neutron energy for different target temperatures, the Doppler broadened cross sections must be obtained first as functions of energy for various target temperatures and then averaged for various neutron energies. For the averaged cross sections presented in figure 8, two fuel temperatures, 0° and $10,000^\circ$ K, were used. The cross sections at energy E for either absorption or fission were computed from the following equation, summed over all resonances:

$$\sigma(E) = \frac{C}{\sqrt{E}} \psi(x,\theta) \tag{1}$$

Here $C/\sqrt{E_o}$ represents the peak resonance cross section for either absorption or fission if no temperature dependence is assumed and C is defined as

$$C = \frac{2.607\times10^6 \Gamma_n \Gamma_\gamma}{\sqrt{E_o}\Gamma^2} g \left(\frac{A + 1}{A}\right)^{3/2} \tag{2}$$

Equation (2) is shown for absorption but can also be written for fission by replacing the partial level width for absorption Γ_γ by the partial level width for fission Γ_f.

$$\psi(x,\theta) = \frac{1}{2\sqrt{\pi\theta}} \int_{-\infty}^{\infty} \frac{e^{-(x-y)^2/4\theta}}{1 + y^2} dy \tag{3}$$

where

$$x = \frac{2}{\Gamma} \frac{A}{A + 1} (E - E_o) \tag{4}$$

and

$$\theta = 3.446\times10^{-4} \frac{E_o T}{A\Gamma^2} \left(\frac{A}{A + 1}\right)^2 \tag{5}$$

Added to the resonance portion of the cross section is a $1/v$ portion that is used to bring the calculated value up to the measured value at an energy of 0.0252 electron volt. The level widths, statistical factors, and measured thermal cross sections were obtained from reference 10.

Average microscopic cross sections were obtained from the defining relation

$$\bar{\sigma} \equiv \frac{\int_0^{E_c} \sigma(E)\varphi(E)\,dE}{\int_0^{E_c} \varphi(E)\,dE} \tag{6}$$

where E_c is the cut-off energy. The energy dependent flux $\varphi(E)$ is given by:

$$\varphi(E) = vn(E) \tag{7}$$

To obtain the neutron number density per unit energy

$$n(E) = \frac{2\pi}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT} \tag{8}$$

it was assumed that the Maxwell-Boltzman distribution applies.

Substituting equations (7) and (8) into equation (6) and simplifying result in

$$\bar{\sigma} = \frac{\int_0^{\infty} \sigma(E)\sigma E e^{-E/kT} dE}{\int_0^{\infty} E e^{-E/kT} dE}$$

The integrations were performed numerically by trapezoidal integration with programs written for the IBM 704 computer.

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TABLE I. - GROUP-AVERAGED MICROSCOPIC CROSS SECTIONS

[Average microscopic cross sections $\overline{\sigma}_a$, $\overline{\sigma}_s$, and $\nu\overline{\sigma}_F$ in barns.]

Group	Energy range	Fuel									
		Uranium 235			Plutonium 239			Deuterium oxide		Graphite	
		$\overline{\sigma}_a$	$\overline{\sigma}_s$	$\nu\overline{\sigma}_F$	$\overline{\sigma}_a$	$\overline{\sigma}_s$	$\nu\overline{\sigma}_F$	$\overline{\sigma}_a$	$\overline{\sigma}_s$	$\overline{\sigma}_a$	$\overline{\sigma}_s$
1	10 to 1.5 Mev	1.34	4.58	3.75	2.00	4.10	6.40	0.00903	6.19	0	1.74
2	1.5 to 0.0043 Mev	1.64	5.98	3.65	1.90	5.72	5.20	0	10.40	0	4.05
3a	0.0043 Mev to 1.37 ev	9.26	15.70	16.26	31.00	12.17	67.20	--------	------	.00010	4.64
3b	0.0043 Mev to 0.125 ev	9.49	15.72	16.82	--------	------	--------	.00006	10.68	.00027	4.67
4a	0.278 ev	156.00	15.70	326.40	1051.00	13.14	1860.00	--------	------	.00088	4.81
4b	0.025 ev	587.00	16.60	1247.00	--------	------	--------	.00087	14.54	.00300	5.20

TABLE II. - NEUTRON BALANCE FOR URANIUM 235 - DEUTERIUM

OXIDE CAVITY REACTOR

[Temperature, 70° F; cavity radius, 150 cm; fuel-region radius, 25 cm; cavity length to diameter ratio, 1.]

(a) With nozzle hole. Fuel density, 2.87×10^{19} atoms per cubic centimeter.

Energy group	Loss due to -			Production
	Absorption		Leakage	
	Fuel	Reflector		
10 to 1.5 Mev	0	2	-----	1
1.5 to 0.0043 Mev	1	0	-----	1
0.0043 Mev to 0.125 ev	3	0	-----	5
0.025 ev	480	61	(453)	993
Totals	484	63	453	1000
	1000			

(b) Without nozzle hole. Fuel density, 2.74×10^{19} atoms per cubic centimeter.

10 to 1.5 Mev	0	2	-----	1
1.5 to 0.0043 Mev	0	0	-----	1
0.0043 Mev to 0.025 ev	3	0	-----	5
0.025 ev	481	65	(449)	993
Totals	484	67	449	1000
	1000			

TABLE III. - NEUTRON BALANCE FOR URANIUM 235 - GRAPHITE CAVITY REACTOR

[Cavity radius, 150 cm; fuel-region radius, 25 cm; cavity length to diameter ratio, 1.]

(a) Temperature, 70° F; fuel density, 28.3×10^{19} atoms per cubic centimeter

Energy group	Loss due to -			Production
	Absorption		Leakage	
	Fuel	Reflector		
10 to 1.5 Mev	4	0	-----	10
1.5 to 0.0043 Mev	7	0	-----	17
0.0043 Mev to 0.025 ev	43	4	-----	76
0.025 ev	434	314	(194)	897
Totals	488	318	194	1000
	1000			

(b) Temperature, 5300° F; fuel density, 16.5×10^{19} atoms per cubic centimeter

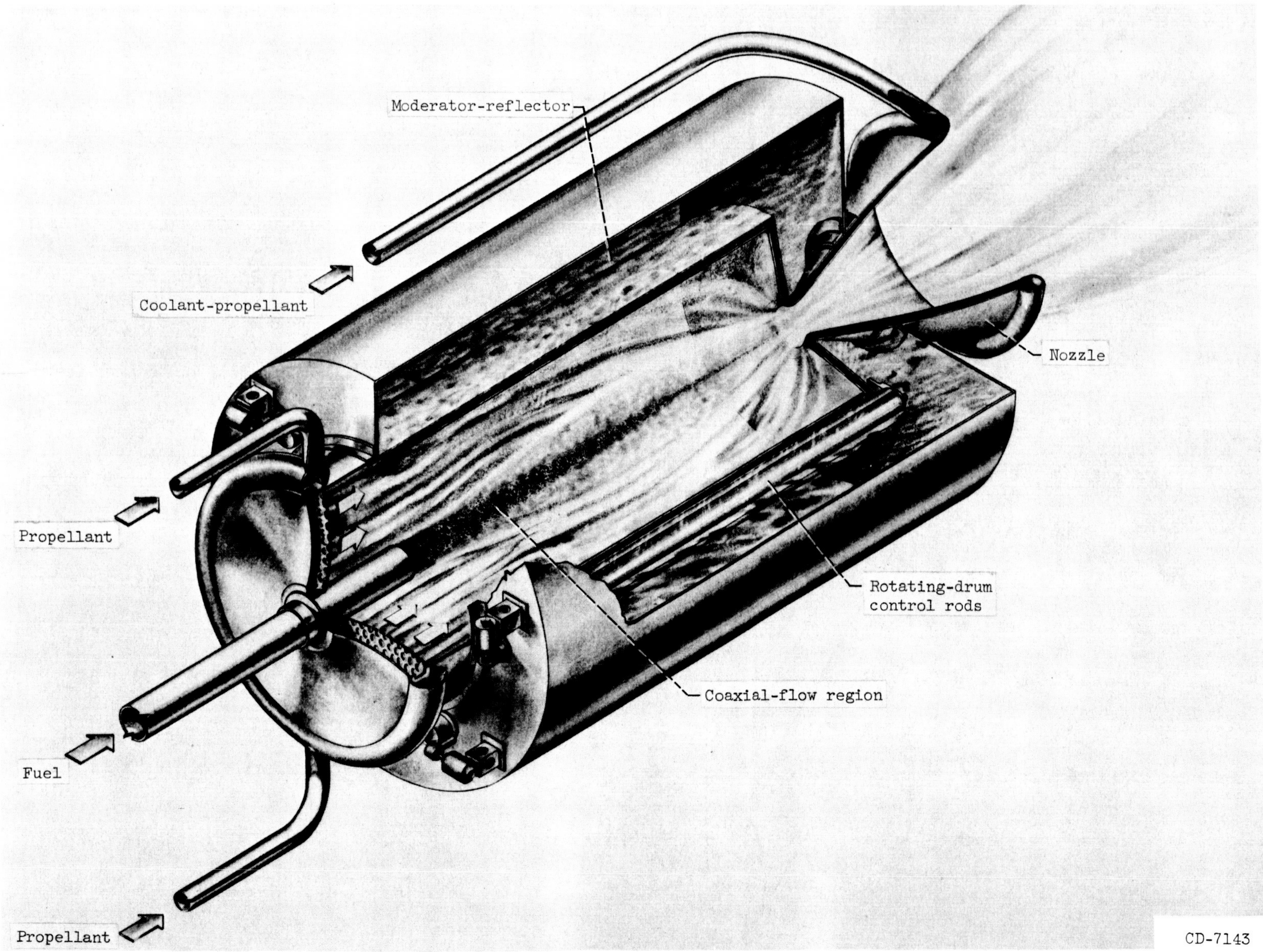
10 to 1.5 Mev	2	0	-----	6
1.5 to 0.0043 Mev	4	0	-----	10
0.0043 Mev to 0.025 ev	26	1	-----	42
0.025 ev	450	143	(374)	942
Totals	482	144	374	1000
	1000			

TABLE IV. - NEUTRON BALANCE FOR PLUTONIUM 239 - GRAPHITE

CAVITY REACTOR

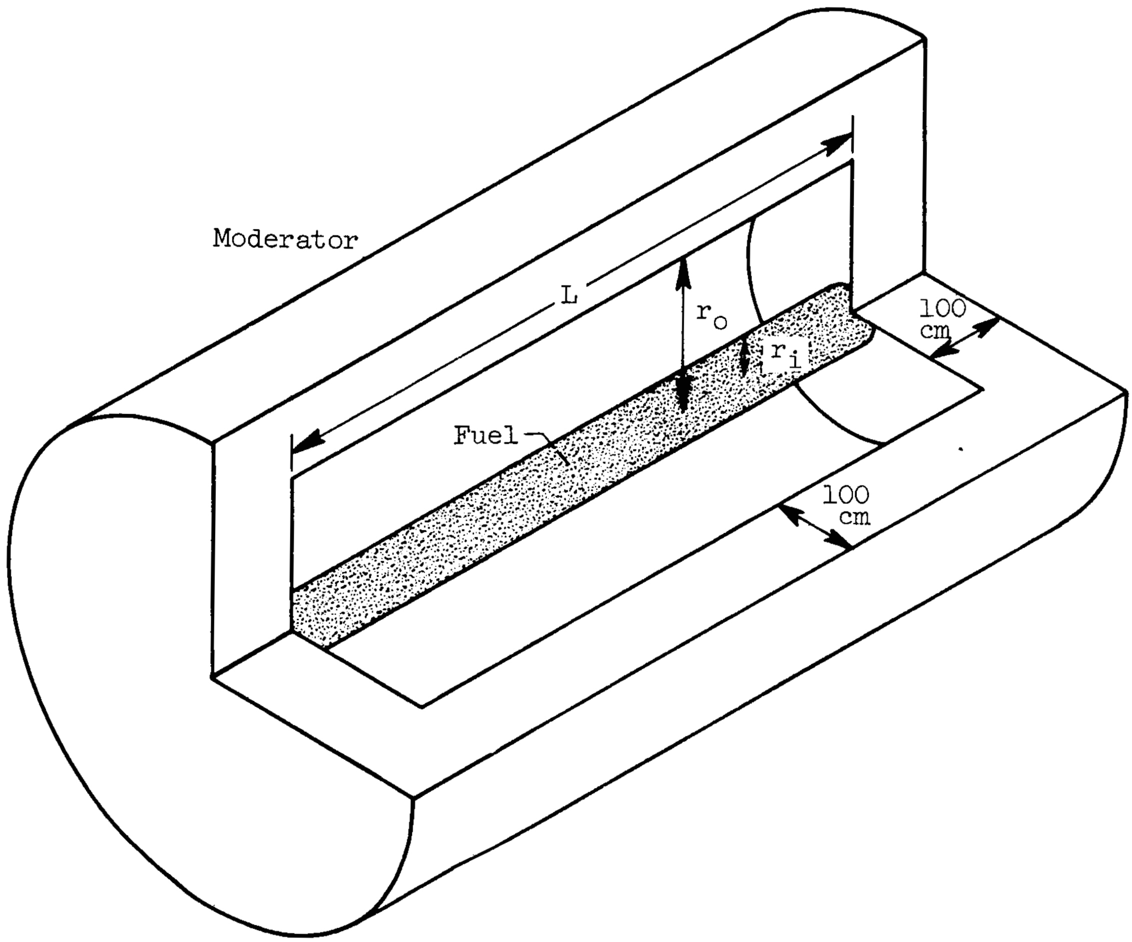
[Temperature, 5300° F; cavity radius, 150 cm; fuel-region radius, 25 cm; cavity length to diameter ratio, 1; fuel density, 11.7×10^{19} atoms/cc.]

Energy group	Losses due to -			Production
	Absorption		Leakage	
	Fuel	Reflector		
10 to 1.5 Mev	2	0	-----	8
1.5 to 0.0043 Mev	4	0	-----	10
0.0043 Mev to 1.37 ev	24	1	-----	52
0.278 ev	523	123	(323)	930
Totals	553	124	323	1000
	1000			



(a) Coaxial-flow gaseous nuclear rocket (ref. 2).

Figure 1. - Gaseous-core cylindrical-cavity reactor.



(b) Basic reactor model for present investigation.

Figure 1. - Concluded. Gaseous-core cylindrical-cavity reactor.

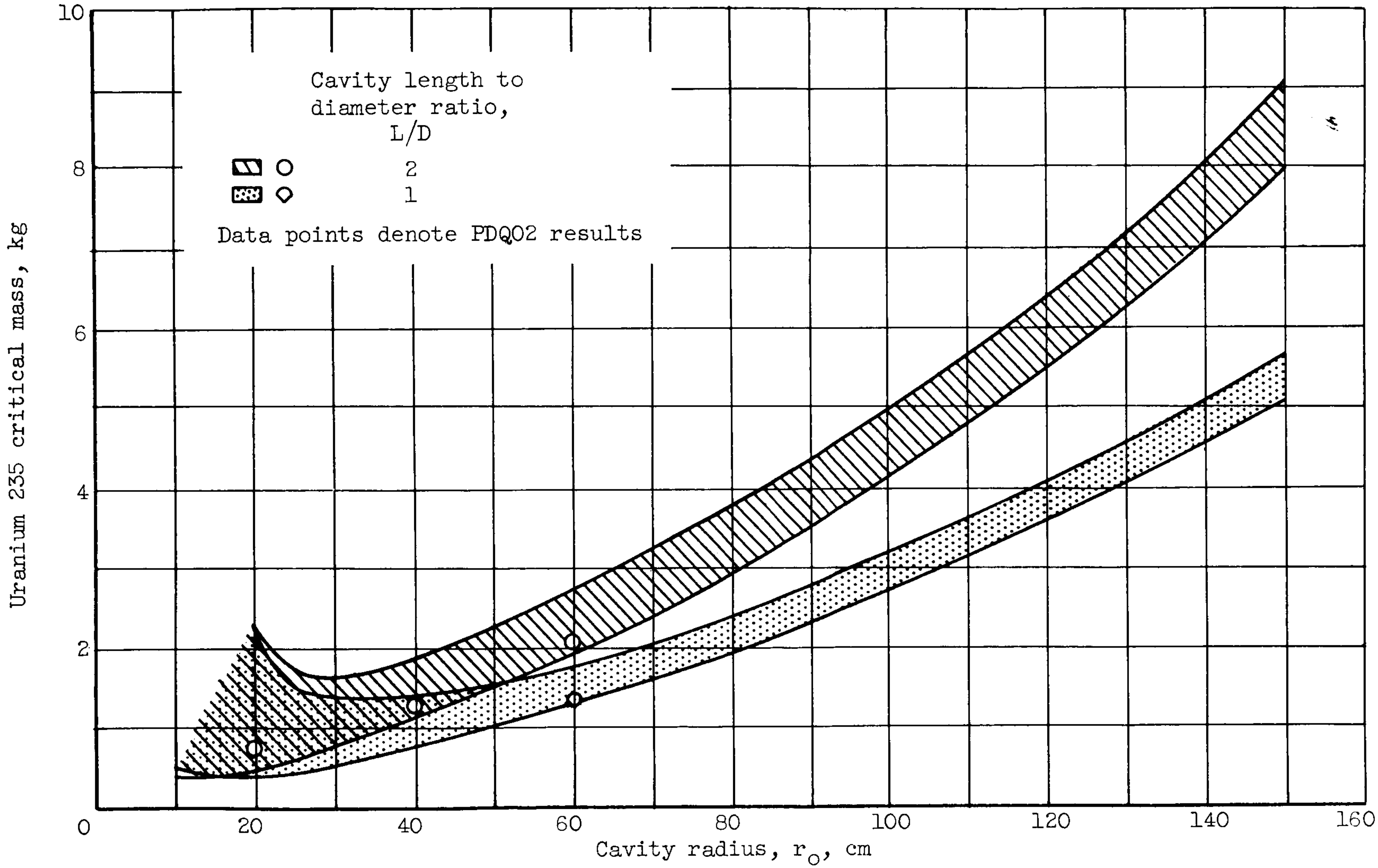


Figure 2. - Critical mass for cylindrical-cavity reactors from buckling analogy. Reflector, deuterium oxide; reflector thickness, 100 centimeters; temperature, 70° F; static criticality factor, 1.

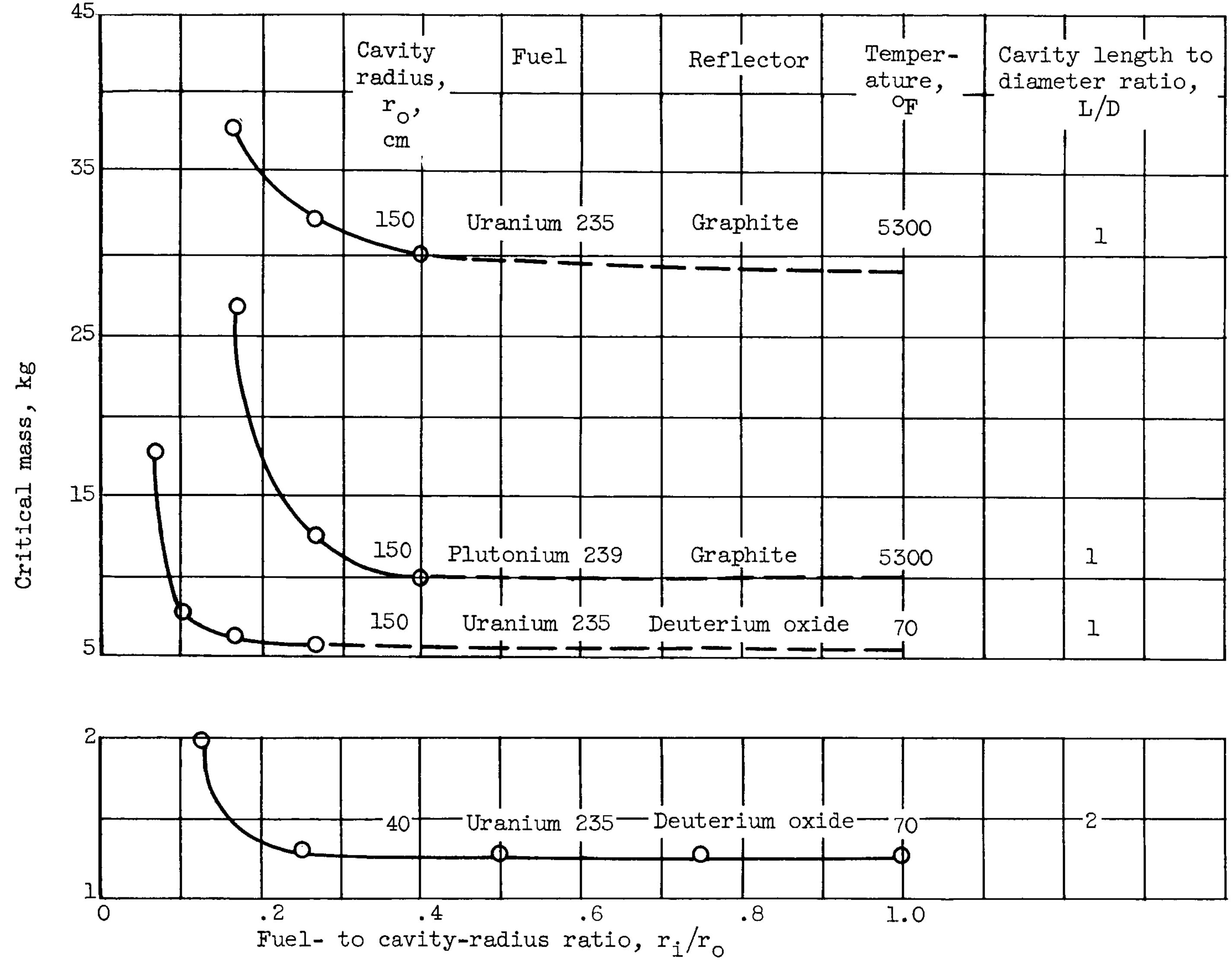


Figure 3. - Effect of decreased fuel-region radius on critical mass of cylindrical-cavity reactors. Two-dimensional PDQ02 results; reflector thickness, 100 centimeters; static criticality factor, 1.

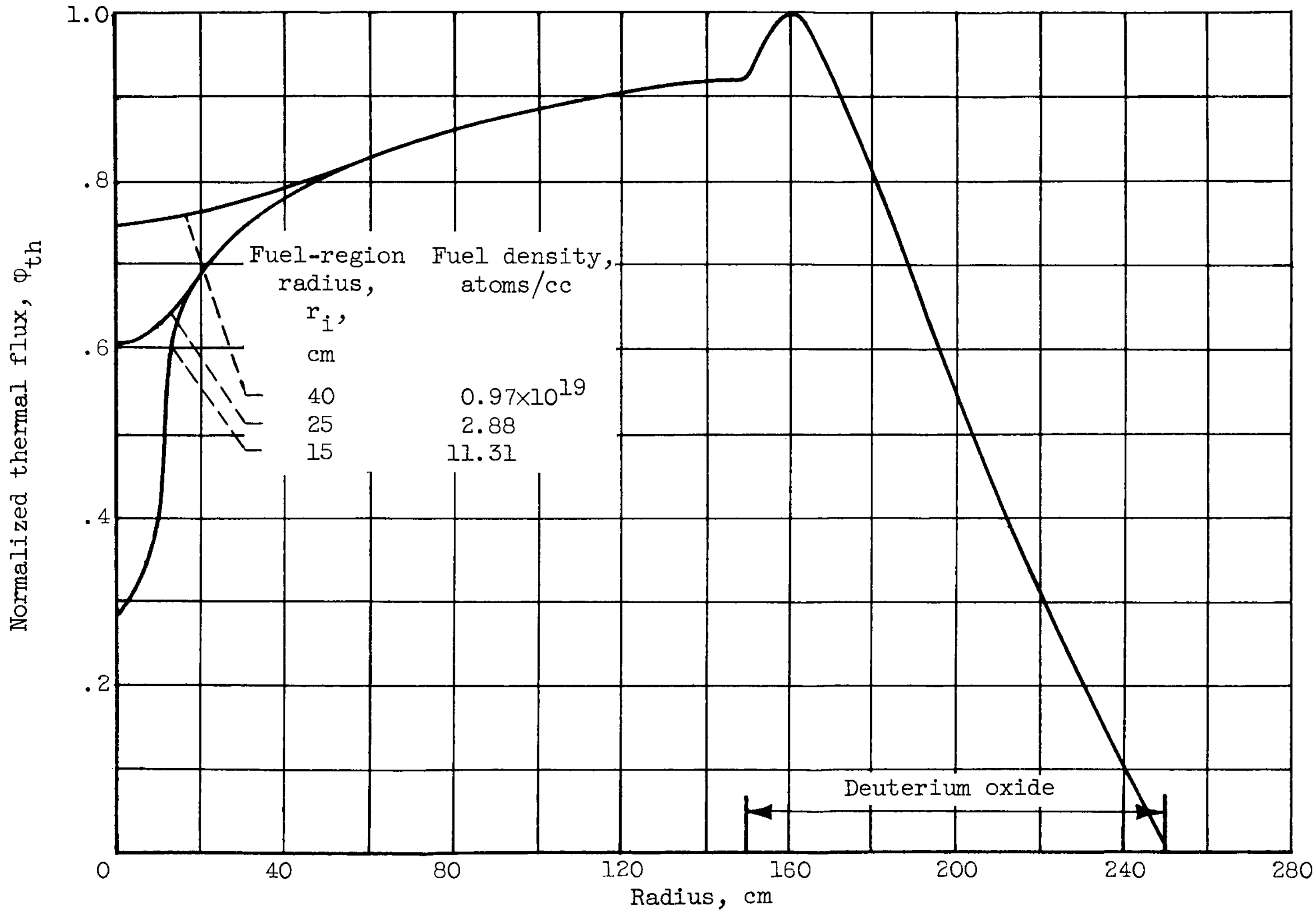


Figure 4. - Effect of thermal flux for reduced fuel-region radii. Fuel, uranium 235; cavity length to diameter ratio, 1; static criticality factor, approximately 1.

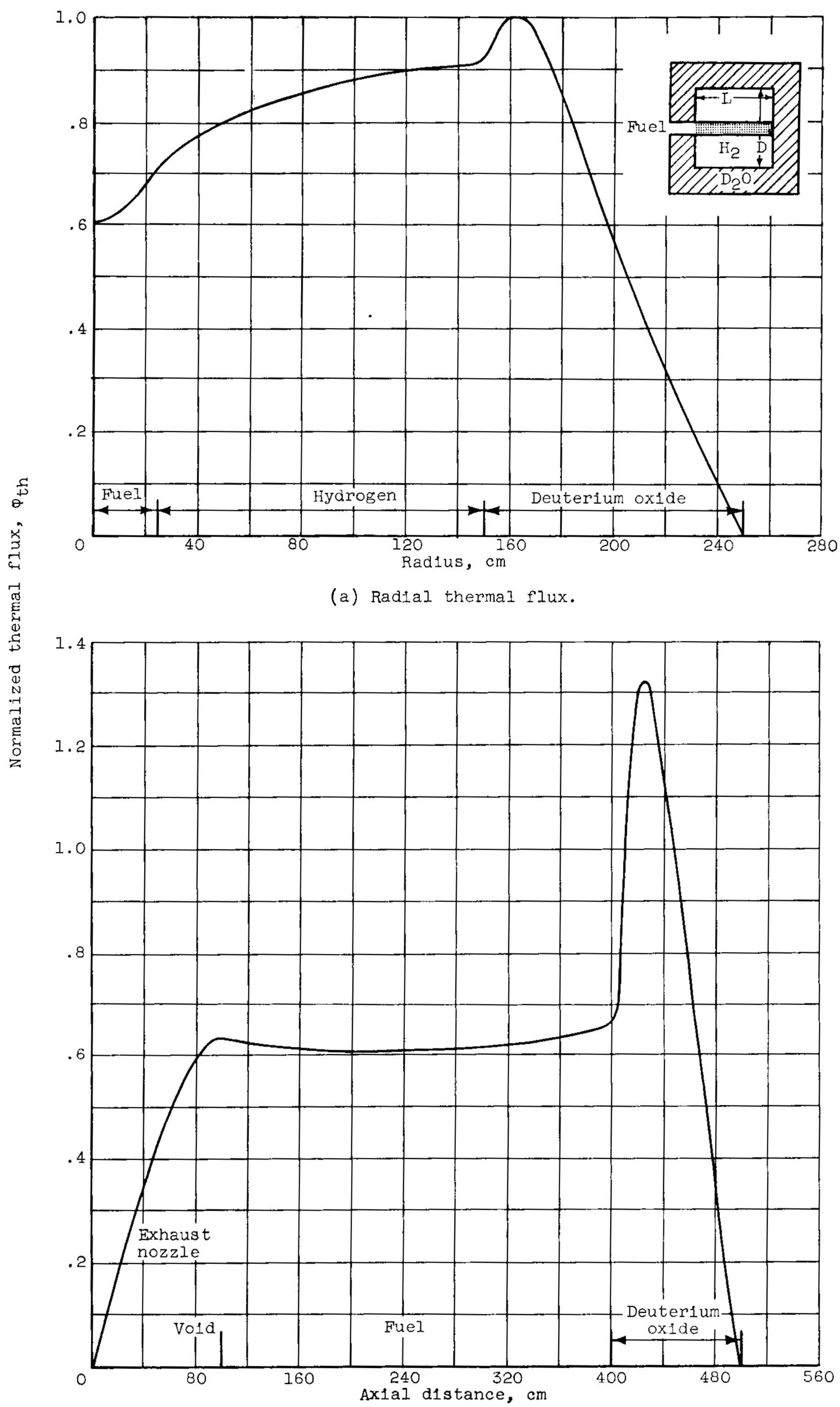


Figure 5. - Thermal flux for nozzle hole in deuterium oxide. Cavity length to diameter ratio, 1; static criticality factor, 1; fuel density, 2.87×10^{19} atoms per cubic centimeter; fuel, uranium 235.

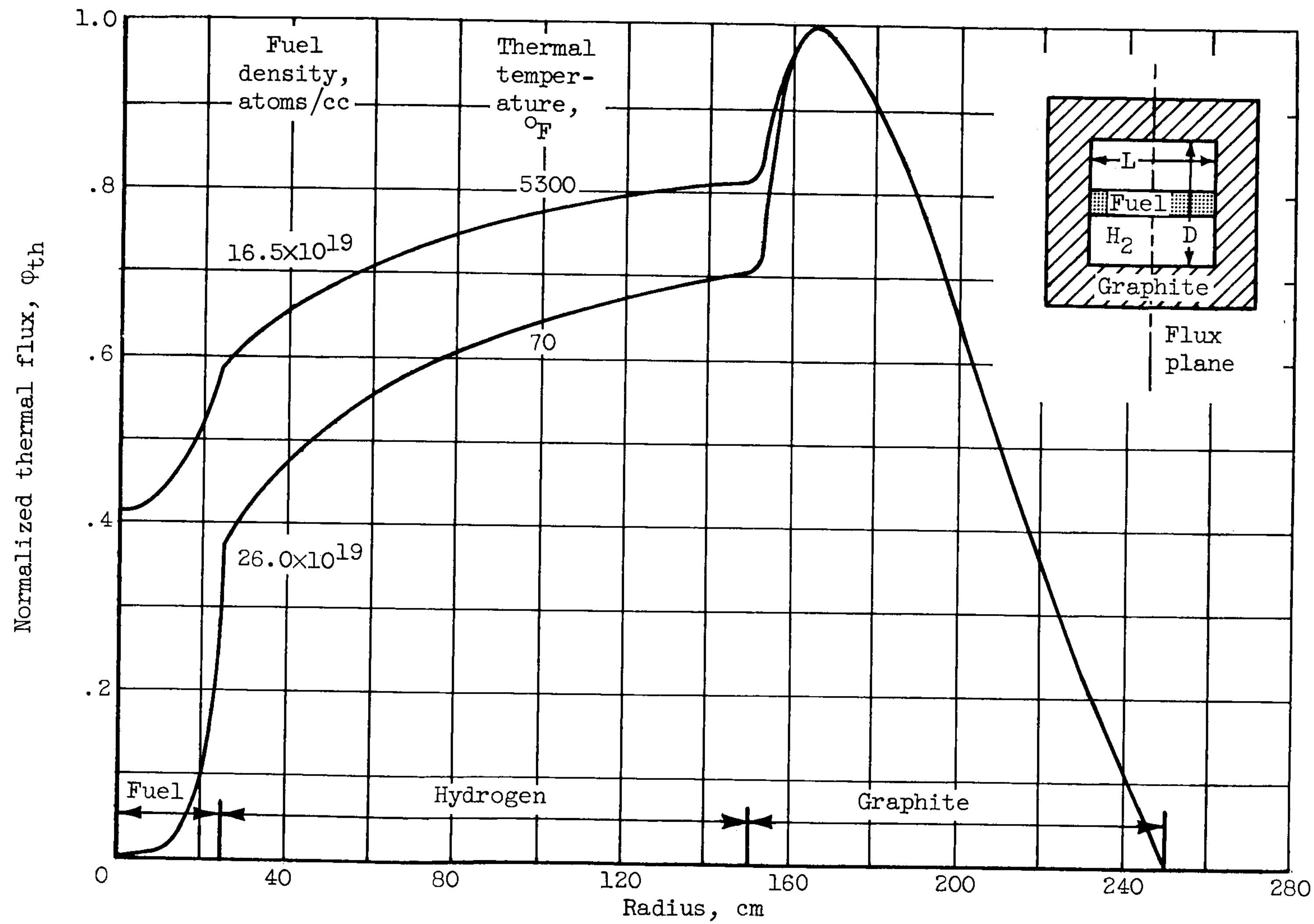


Figure 6. - Thermal neutron flux for graphite reflected cavity at various temperatures. Cavity length to diameter ratio, 1; static criticality factor, 1; fuel, uranium 235.

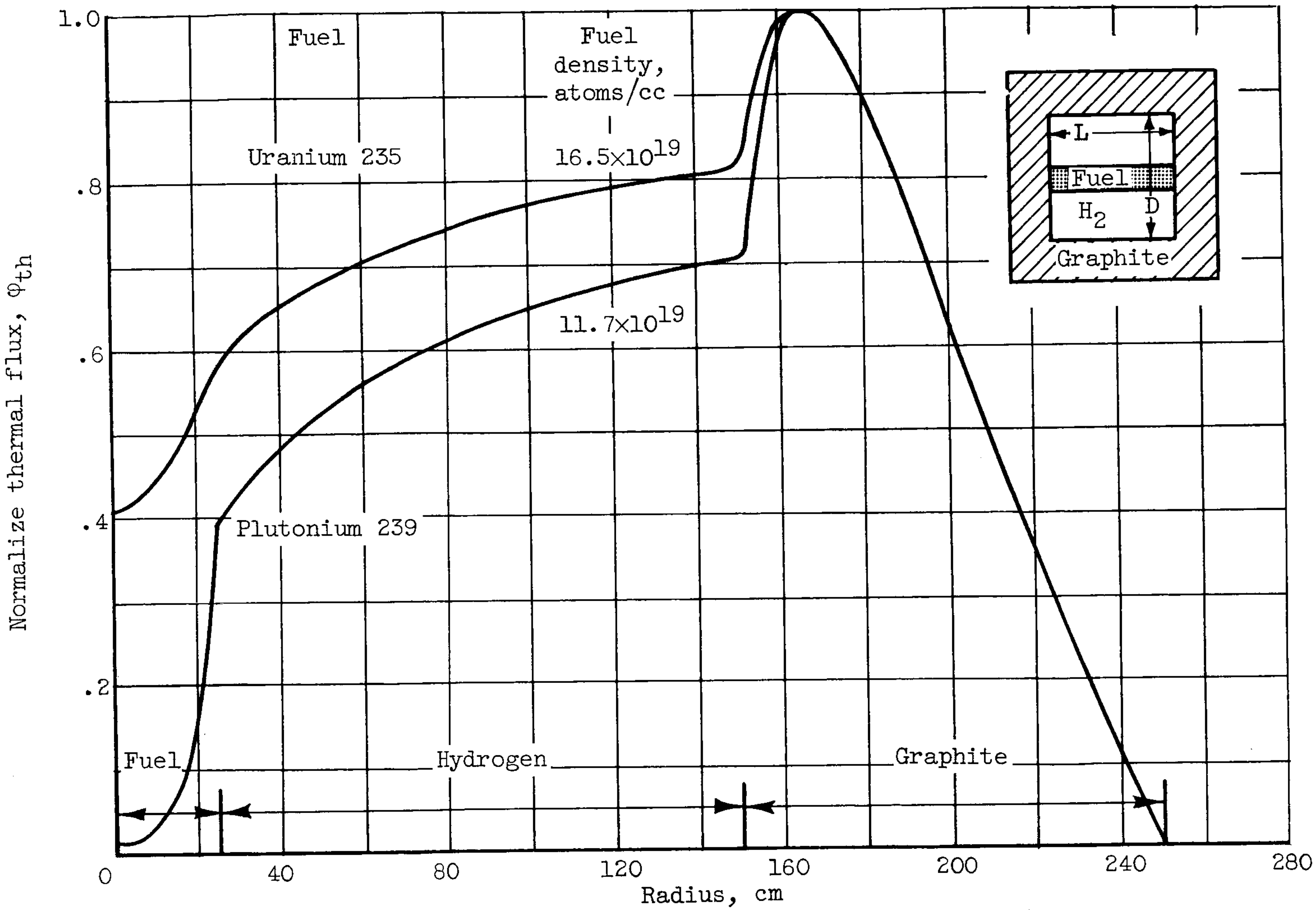


Figure 7. - Thermal neutron flux for graphite reflected cavity with various fission fuels. Temperature, 5300° F; cavity length to diameter ratio, 1; static criticality factor, 1.

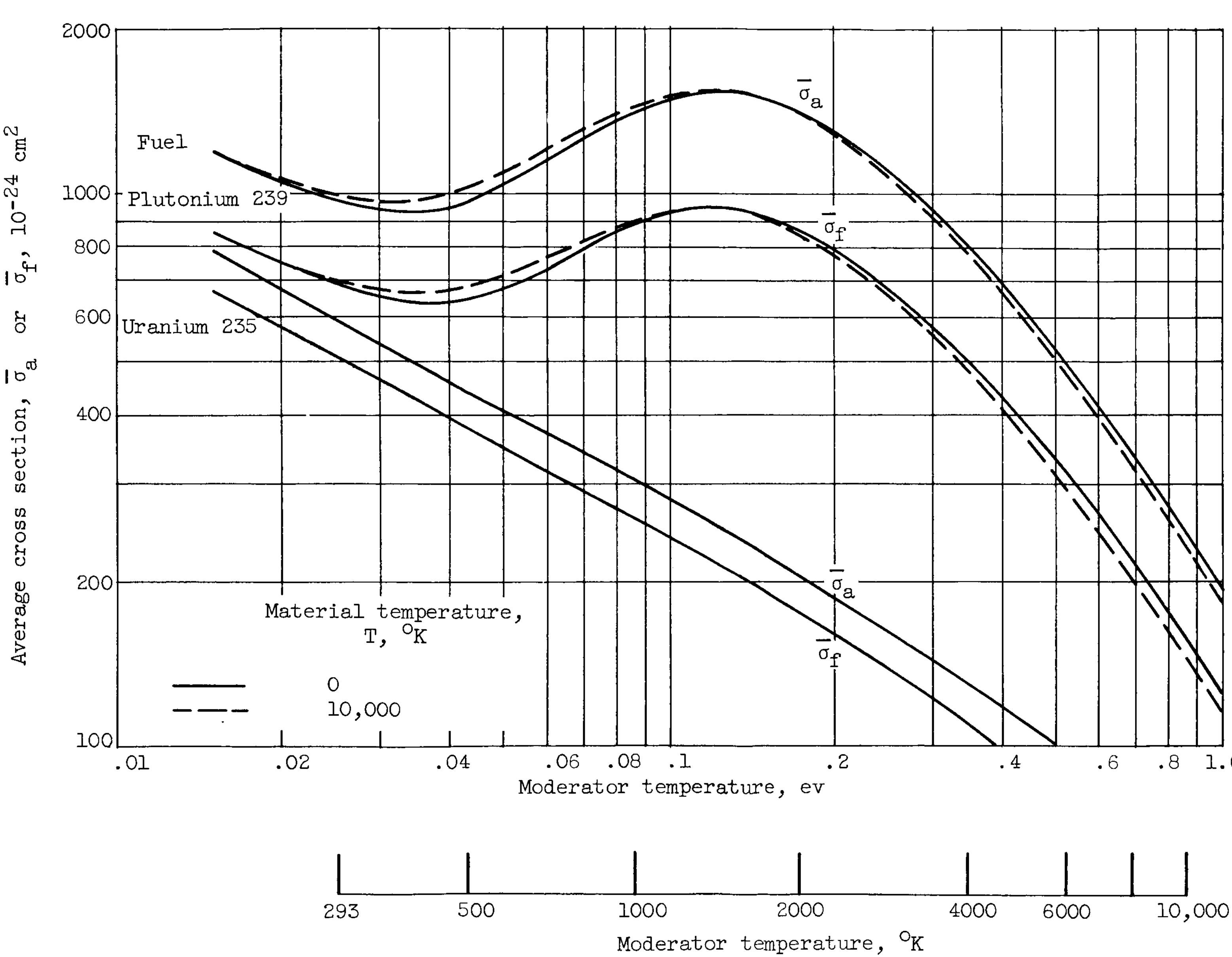


Figure 8. - Average neutron cross sections for plutonium 239 and uranium 235.

NASA-Langley, 1963 E-1738